3D Bedrock Channel Evolution with Smoothed Particle Hydrodynamics Coupled to a Finite Element Earth

Nick Richmond[1], Peter Koons[1,2], Phaedra Upton[3]
[1]University of Maine, School of Earth and Climate Sciences, Orono, Maine, USA
[2]University of Maine, Climate Change Institute, Orono, Maine, USA
[3]GNS Science, Lower Hutt, New Zealand
Contact: nicholas.richmond@maine.edu

Abstract

An enduring obstacle to reliable modeling of the short and long-term evolution of the stream channel-hillslope ensemble has been the difficulty of estimating stresses generated by stream hydrodynamics. To capture the influence of complex three-dimensional (3D) flows on bedrock channel evolution, we derive the contribution of hydrodynamic stresses to the stress state of the underlying bedrock through a Smoothed Particle Hydrodynamics (SPH) approximation of the Navier-Stokes equations as calculated by the DualSPHysics code (Crespo et al., 2015). Coupling the SPH flow solutions to the stress-strain formulation of the Failure Earth Response Model (FERM) (Koons et al., 2013) provides three-dimensional erosion as a function of the strength-stress ratio of each point in the computational domain. From the coupling of SPH and FERM we gain a 3D physics-based erosion scheme and a two-way link between complex flows and hillslope dynamics in a finite element framework.

Motivating Questions:

1.) What are the hydraulic stresses generated in bedrock channels, and how do they vary in time and space?
2.) What is the geomorphic response to the hillslope stresses and hydraulic stresses in a bedrock channel with heterogeneous rock strength?

Methods

Smoothed Particle Hydrodynamics (SPH)

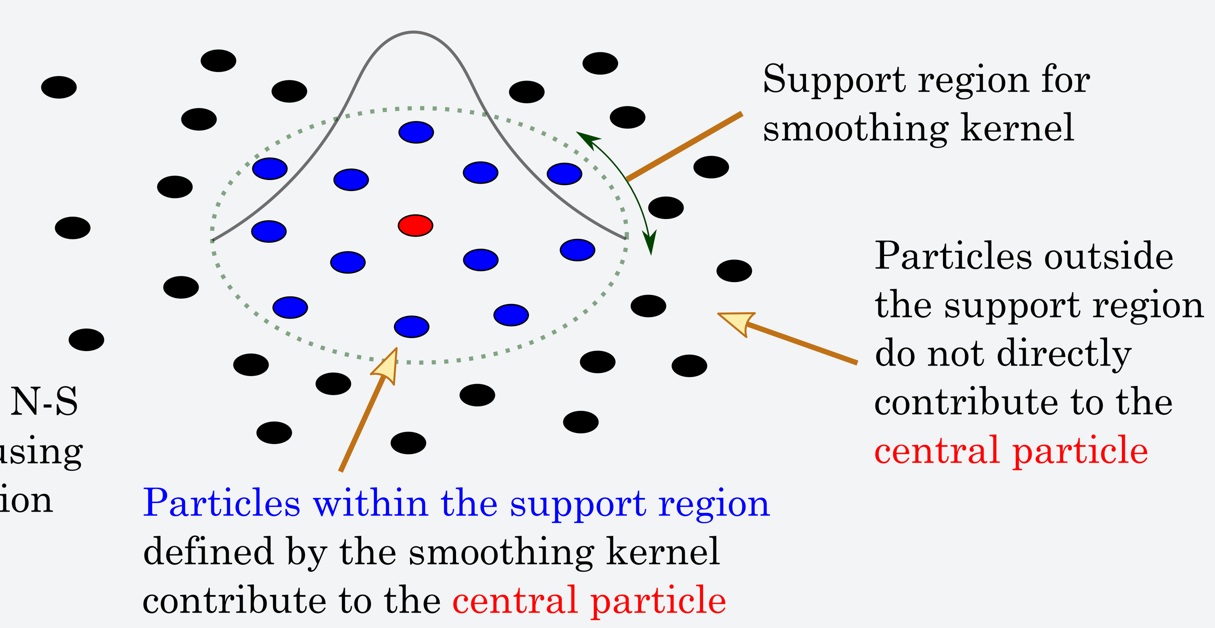
The physics of motion for fluids (Navier-Stokes [N-S] equations, shown below in simplified notation) are solved for each particle at every timestep.

A Basic SPH Smoothing Kernel
(after Karekal, Das, Mosse, & Cleary, 2011)

Navier-Stokes
(Simplified Notation)

$$\rho\frac{\partial \vec{v}}{\partial t} = \nabla P - \rho g + \mu \nabla^2 \vec{v}$$

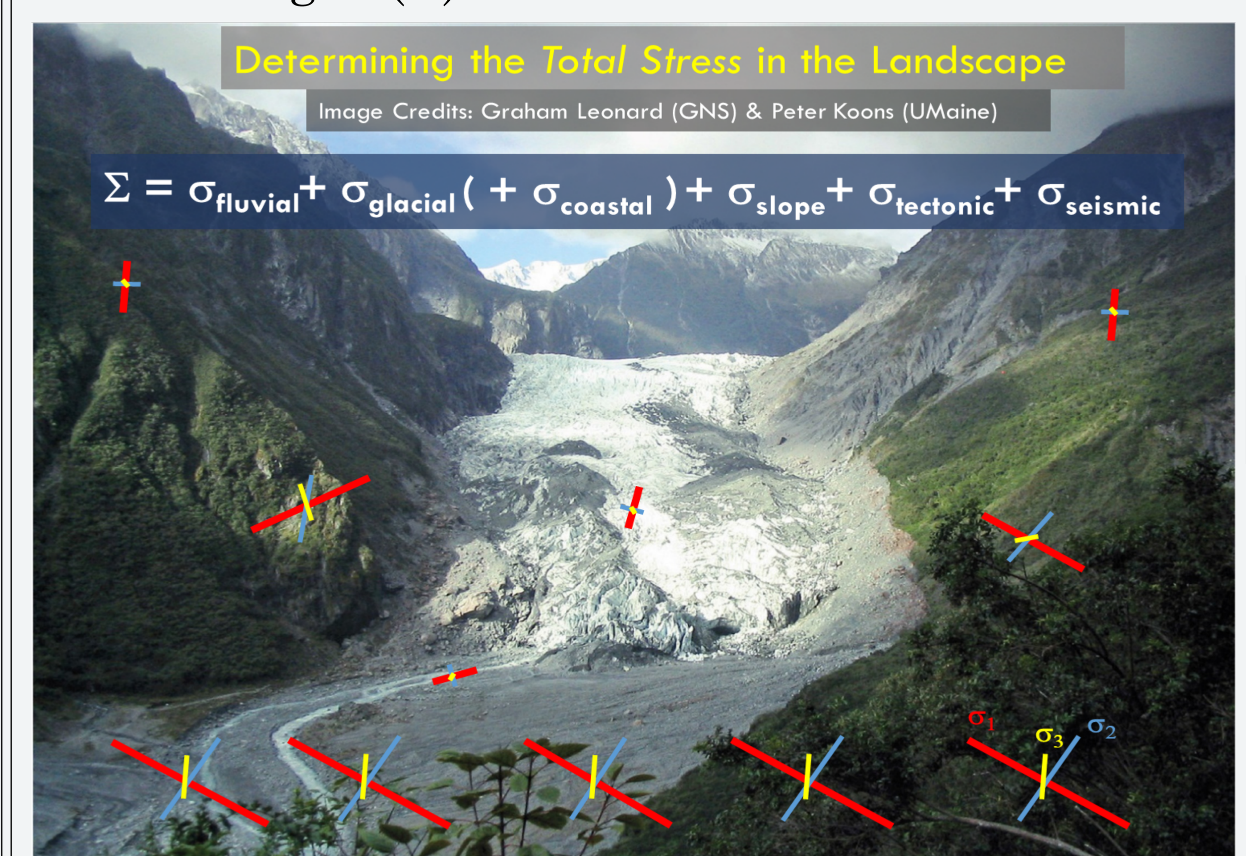
For a given particle, (such as the central particle shown right), the N-S equations are locally integrated using the position and motion information of neighbor particles.

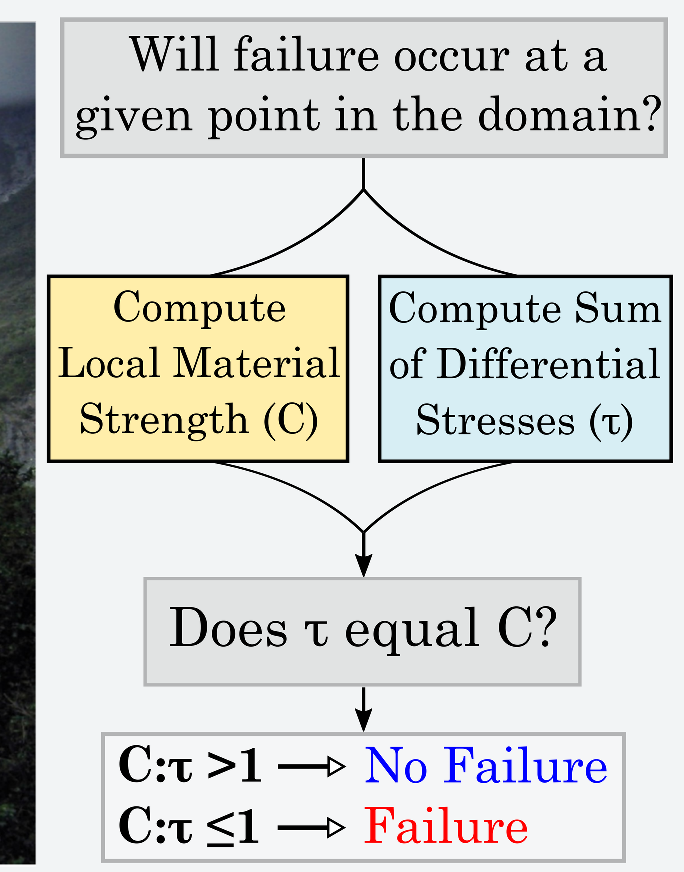


SPH is able to handle the fluid accelerations very well, which allows for realistic simulation of natural flows and fluid-structure interaction.

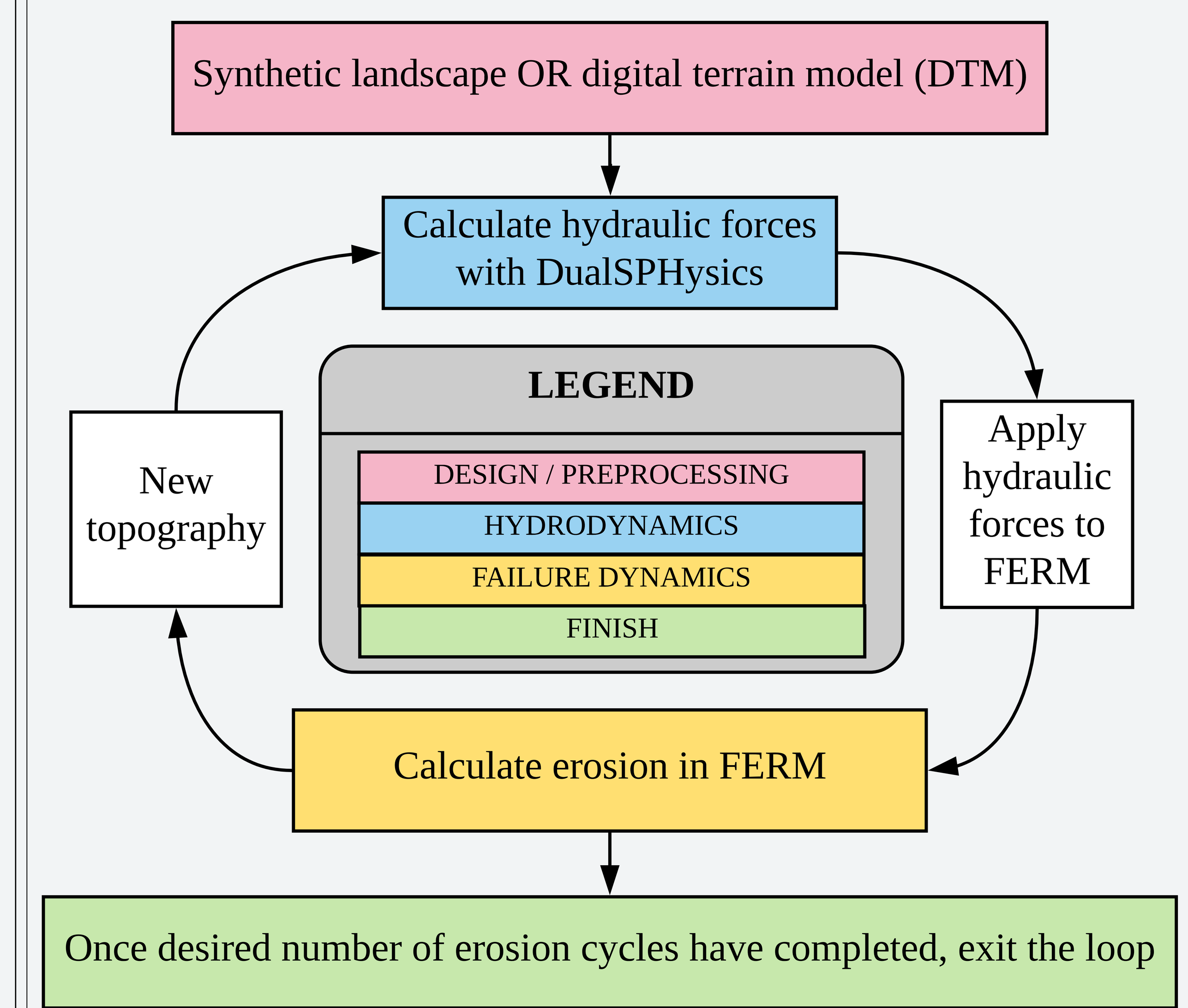
Failure Earth Response Model (FERM)

FERM uses a Mohr-Coulomb approach to failure of Earth materials wherein failure occurs if the local differential stress (τ) exceeds the local strength (C) of the Earth material.

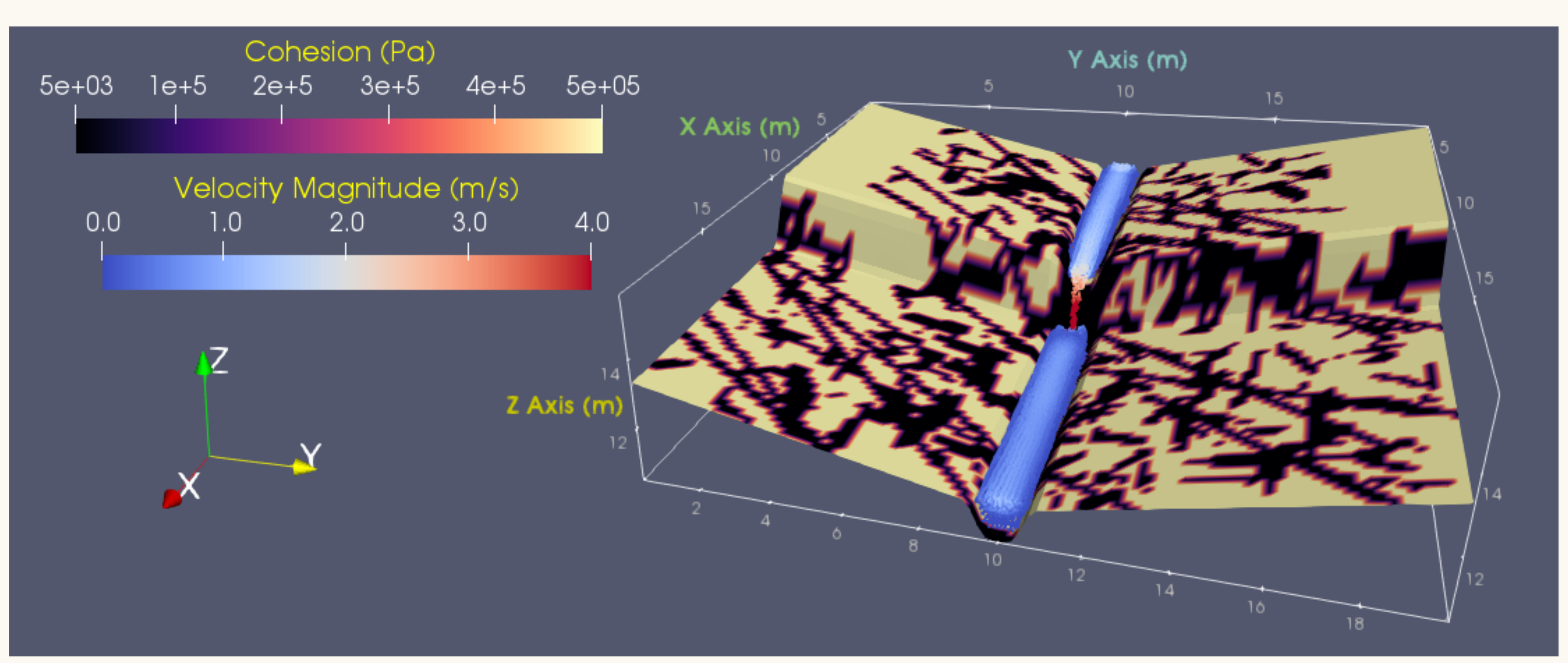




Coupling SPH With FERM

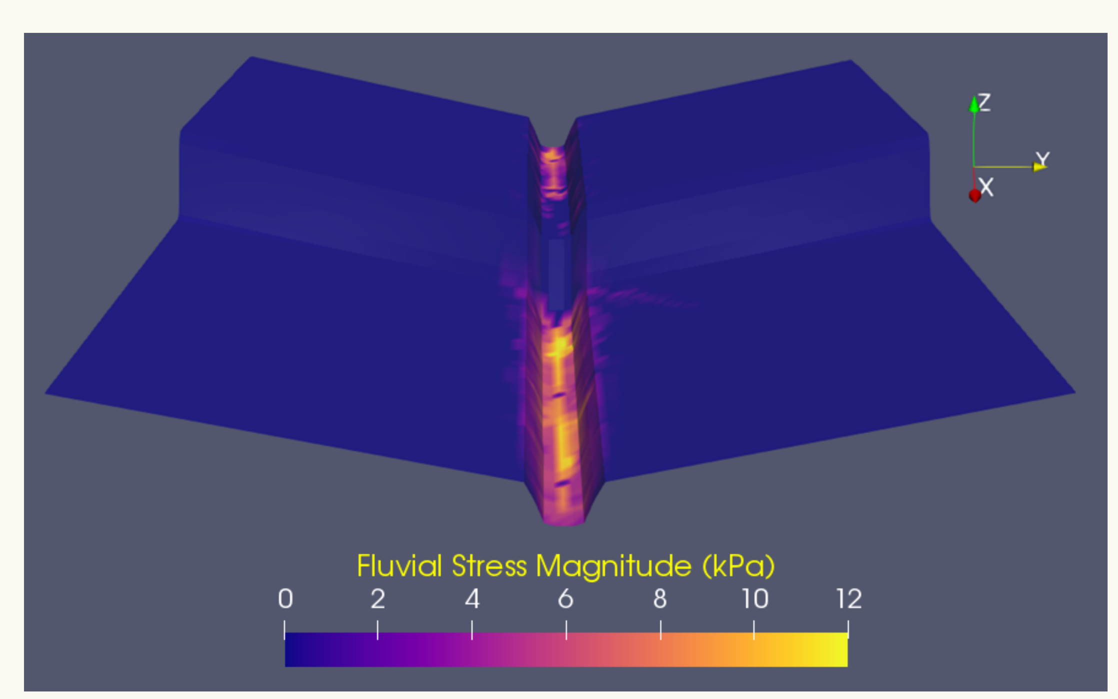


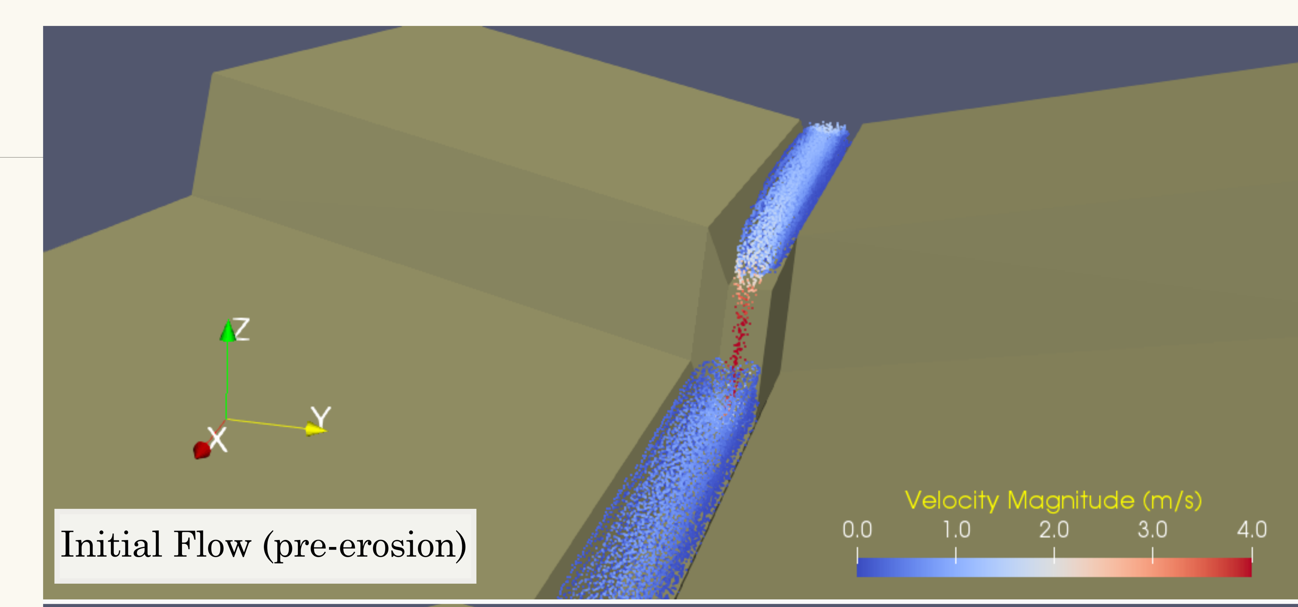
Results

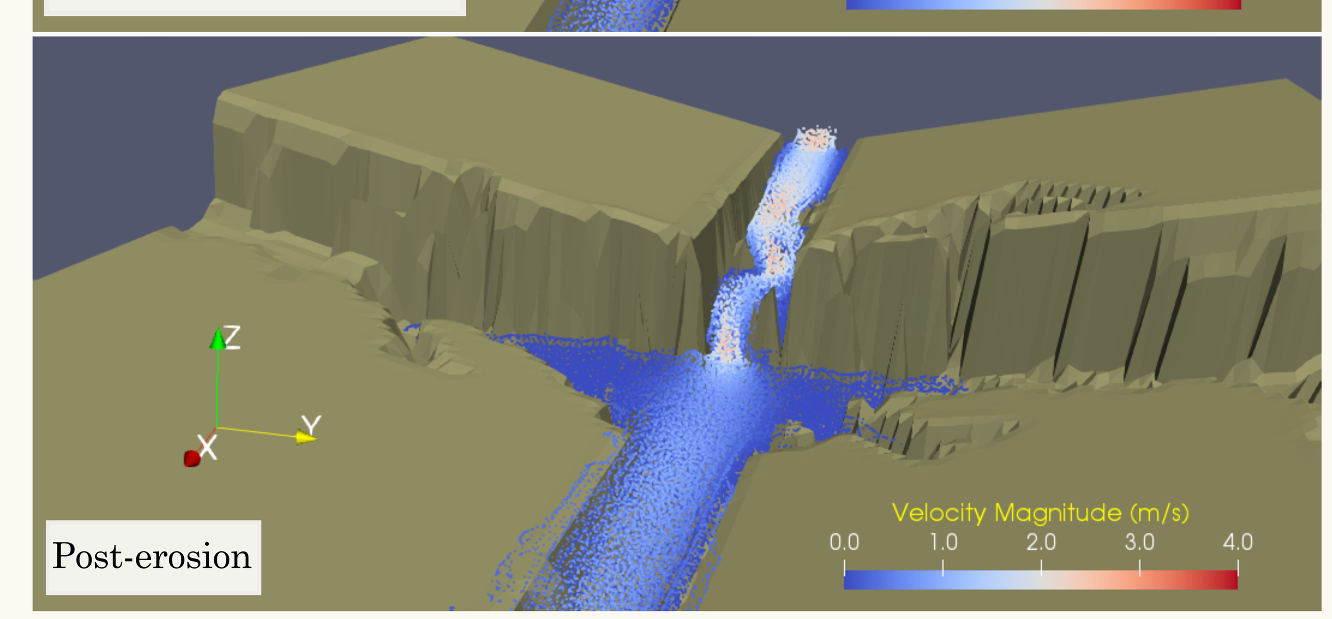


In the above synthetic bedrock channel (20 m * 20 m with a 2 m vertical drop at the knickpoint), water flows over bedrock with strength heterogeneities which are defined in space by a power law fracture distribution. The small waterfall marks a transition in hydraulic stresses from shear-dominant to normal-dominant.

Up to 12 kPa of hydraulic stress is generated in the channel and is highest where the water impacts normal to the channel bed. This suggests that the capacity for bedrock channel incision is high where local flow accelerations dominate.







Erosion via coupled FERM-DualSPHysics modeling shows a reorganization of the channel's velocity structure as weak material is preferentially removed from the domain.

Note the development of a series of meanders on the upstream side of the waterfall.

Acknowledgements

A.J.C. Crespo, J.M. Domínguez, B.D. Rogers, M. Gómez-Gesteira, S. Longshaw, R. Canelas, R. Vacondio, A. Barreiro, O. García-Feal, Comput. Phys. Commun. 187 (2015) 204–216.
P.O. Koons, P. Upton, S.G. Roy, G.E. Tucker, in:, AGU Fall Meet., American Geophysical Union, San Francisco, 2013.
S. Karekal, R. Das, L. Mosse, P.W. Cleary, Int. J. Rock Mech. Min. Sci. 48 (2011) 703–711.

Support from NSF GeoPRISMS Grant OCE-1249909, NSF GLD 1324637, and NSF CDI 1027809 grant to Peter Koons, NH EPSCoR #7552151 (Sean Smith), and University of Maine Graduate School Government

Conclusions and Future Work

The coupling of a particle-based flow solution with a finite element geomorphic solution is a novel approach to bedrock channel incision which robustly approximates the 3D stresses and geomorphic response of bedrock channels with complex geometries and lithologies. Future development of this model will focus on real-time cosimulation of failure and flows in order to capture the complex dynamics of sediment transport and the erosional effects of mobile sediment in bedrock channels and surrounding hillslopes.